

# An Intertemporal Capital Asset Pricing Model With Owner-Occupied Housing

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## Abstract

This paper studies portfolio choice and asset pricing in the presence of owner-occupied housing in a continuous time framework. I show that the market portfolio is not mean-variance efficient, and traditional CAPM fails in a model with owner-occupied housing as both a consumption good and a risky asset when covariances between housing and other risky assets are not zero; however, a conditional linear factor pricing model can still be derived. In this model the market portfolio return and housing return are two pricing factors. Moreover, the non-durable consumption to housing ratio,  $ch$ , is shown to affect expected returns in the same way as  $cay$  (the consumption-to-wealth ratio in Lettau and Ludvigson 2001a, 2001b). The empirical evidence shows that  $ch$  can predict asset returns.  $ch$  is also shown to enter linearly the stochastic discount factor of the economy. The cross-sectional Fama-MacBeth regressions show that the conditional models conditioning on  $ch$  perform much better than their unconditional counterparts, and the conditional two factor model derived in this paper performs almost as well as Fama-French three-factor model.

Keywords: Intertemporal Asset Pricing, Owner-Occupied Housing, Factor Pricing Model

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# 1 Introduction

Owner-occupied housing is the single most important consumption good as well as the dominant asset in most households' portfolios. The demand for owner-occupied housing is thus a combination of intratemporal consumption choice and intertemporal portfolio choice. The dual role of owner-occupied housing should have at least the following two effects on asset pricing. First it changes the market portfolio, and thus changes CAPM; second, owner-occupied housing changes the marginal utility of nondurable consumption if the utility function is nonseparable in nondurable consumption and housing; it therefore changes the consumption based CAPM. I study the cross-sectional implications of owner-occupied housing by combining these two effects in this paper.

Merton (1973) shows that the market portfolio of financial assets is mean-variance efficient with a time-invariant investment opportunity set. He did not allow for the possibility that one of the assets enters the utility function as a consumption good. How does owner-occupied housing change the characteristics of the market portfolio? Does two-fund separation still hold? If it holds, under what specific assumptions? Is wealth portfolio mean-variance efficient? Can housing or real estate risk purely be taken as a linear factor in factor pricing models? I show the following in this paper: first, in general, two fund separation does not hold, and the wealth portfolio is not mean-variance efficient, although the conditional linear factor pricing model still holds, in which the market portfolio return and the return on housing are two pricing factors; second, since housing demand is a combination of consumption demand and asset demand, it contains information about the expected returns of traded assets. Specifically, the non-durable consumption to housing stock ratio, which I call  $ch$  hereafter, enters linearly the stochastic discount factor, and thus can predict returns of risky assets.

This model also shows that the nondurable consumption to wealth ratio, which was called  $cay$  in Lettau and Ludvigson (2001a, 2001b), can also predict the asset returns in the same way as  $ch$ . Lettau and Ludvigson (2001a, 2001b) show empirically that  $cay$

predicts asset returns and conditional versions of (C)CAPM conditioning on *cay* perform much better than their unconditional versions. They made an assumption that *cay* enters linearly the stochastic discount factor of the economy to carry out their unconditional tests. In the model I propose, I show explicitly that the consumption-to-wealth ratio enters linearly the stochastic discount factor. Moreover, a major criticism of Lettau & Ludvigson (2001) is that the fact that *cay* predicts market returns cannot justify the use of *cay* as the only conditional variable. We need to know the full information set of investor to carry out the test. While in my model, I derived an exact stochastic discount factor that enables me to test the model even without knowing the full information set.

This paper is related to three broad streams of literature. The first is intertemporal asset pricing model literature as in Merton (1973). This paper modifies Merton's ICAPM by introducing owner-occupied housing. While Merton obtains the nice analytical result of a multi-factor pricing model, it is not obvious what exactly the second factor is other than the market portfolio return. This paper also derives a multi-factor pricing model based on plausible assumptions; however, the factors in this model are concrete, easier to measure, and the empirical tests of this model are easier to implement. A unique feature of this model, as compared to Merton (1973) is that there are two consumption goods, and one of them is also an asset, which complicates the asset pricing implications of the model. By adding owner-occupied housing into the classical analysis, the model provides more insights into asset pricing, since owner-occupied housing is different from any other consumption good as well as any other risky financial asset.

The second stream of literature concerns portfolio choice and asset pricing in the presence of housing. In their seminal work, Grossman and Laroque (1990) first examined this problem in a continuous time framework. Their results rely on two simplifying assumptions: (1) they abstract totally from non-durable consumption, agents care only about the housing consumption, and (2) they assume house prices are constant. Based on these assumptions, they conclude that even in the presence of durable consumption

goods, two-fund separation still holds, and the market portfolio is mean-variance efficient. They made these two assumptions mainly because they want to explicitly characterize the  $(s,S)$  adjustment rule of housing in the presence of adjustment costs. In a more recent paper, Flavin and Nakagawa (2004) relax the above two assumptions and focus on the effect of adjustment costs on the equity premium. In their model, both non-durable consumption and housing enter the utility function in a non-separable way, and the house price is explicitly stochastic and follows geometric Brownian motion. Flavin and Nakagawa also conclude that the market portfolio is mean-variance efficient and the traditional CAPM holds. They assume that the covariance matrix of the asset returns (including housing return) is block diagonal. In this paper, as Flavin and Nakagawa, I model both housing and non-durable consumption and I allow house price to follow a diffusion process. However, I remove the block-diagonal covariance matrix assumption. The reason is that even if the stock market shows little covariance with housing, it is not the case that every stock shows little covariance with housing. As mentioned earlier, this change in the assumption reverses some of the results in the papers mentioned above.

Piazzesi et al. (2006) developed a representative agent consumption-based asset pricing model with housing; they obtain similar results to those in this paper. For example they also find that the composition ratio of nondurable consumption to housing is a pricing factor in addition to the consumption growth rate in a consumption based asset pricing framework. Contrary to Piazzesi et al, I do not allow homeowners consume an amount of housing different than what they own. Another important difference here is that I allow agents to differ in their wealth. They also focus on producing predictability in excess return related to housing variables. I also establish the predictability results in this paper, but I focus more on the cross-sectional implication of the model. Notice also that although my asset pricing results are derived not in a representative agent model, the model can aggregate into a representative agent framework under my assumptions<sup>1</sup>.

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<sup>1</sup>The author thanks Monika Piazzesi for pointing this out.

I don't use the representative agent framework because I also want to establish the n-fund separation results, which is meaningless in a representative agent framework. Davis & Martin (2006) estimate the parameters of Piazzesi et al's model, and show that the model cannot match house price, stock price and T-bill return at the same time for plausible parameter values. They conclude that the housing model cannot resolve the equity premium puzzle.

Chetty and Szeidl (2005) focus on the fact that housing consumption can only be adjusted infrequently, and show that the housing commitment mechanism can be a rational explanation for the habit formation model, which can potentially resolve the equity premium puzzle. But they ignore the fact that housing is also an asset, and the model again aims at resolving the equity premium puzzle to some extent. In this paper, however, I am interested in investigating the cross-sectional implications of housing on asset pricing. Lustig and Van Nieuwerburgh (2006) use another mechanism, i.e. the collateral constraints mechanism, to study the implication of housing on asset pricing. They show that the asset prices are closely related to the collateral ratio, because the collateral ratio affects households' exposure to idiosyncratic risk, and thus risk sharing of the whole economy. In their (2005) paper, they show empirically that the collateral ratio can predict expected returns, and can be used as a conditioning variable in cross-sectional asset pricing test.

The third stream of literature introduces real estate risk as a common risk factor in asset pricing models. The basic rationale is that real estate composes a large proportion of national wealth; thus based on the logic of the traditional CAPM, the wealth portfolio should include real estate. Kullmann(2003) examined the performance of the factor pricing model by introducing real estate risk as an independent risk factor. She finds that the inclusion of real estate risk can greatly improve the performance of factor pricing models in terms of the explanatory power for cross-sections of stock returns. However, theoretically, she does not discuss how the dual role of housing can affect the CAPM

model. This paper, however, provides a theoretical foundation by showing that the conditional linear factor pricing model, with real estate risk as one of the pricing factors, holds, although the traditional CAPM fails.

In summary, this paper develops an asset pricing model with owner-occupied housing in a continuous time framework. The driving forces of the model are: 1) The dual role of housing as a consumption good and a risky asset; 2) General covariance matrix between housing and stocks; 3) A Cobb-Douglas aggregate of utility function is employed to get a simplified conditional asset pricing model. The main results include the following: first, both two-fund separation and CAPM fail with owner-occupied housing ; second, both nondurable consumption-to-wealth (*cay*) and non-durable consumption to housing ratio (*ch*) enter the stochastic discount factor linearly. (The Cobb-Douglas form of utility function is crucial for linearity here). The first result, which relies only on the first two assumptions, can be derived in a one period mode. However, dynamics is extremely important for the second result, where the asset pricing dynamics are related to the dynamics of *cay* and *ch*.

The rest of the paper is organized as follows: section II elaborates the basic household problem and solves for the consumption and portfolio choice in a general setting. Section III develops the equilibrium asset pricing model. In section IV, I test the empirical performance of the model and compare it with other benchmark empirical models, namely Fama-French three factor model and Kullmann's real estate pricing model.

## 2 Basic Model

### 2.1 Model Setup

The economy is populated by  $K$  infinitely lived agents, who consume a composite non-durable consumption good and owner-occupied housing. The agents can trade three

different types of assets. The first type is a riskless asset, the second type is  $n$  risky assets, and the third type is owner-occupied housing. For the purpose of this paper, I assume that all these three types of assets can be traded without transaction costs and the capital market is structured in exactly the same way as in Merton (1973)<sup>2</sup>.

Each agent maximizes the expected utility of the form:

$$\max_{\{C^k, H^k, \alpha^k\}} E \left\{ \int_0^\infty e^{-\delta t} U^k(C_t^k, H_t^k) dt \right\} \quad (1)$$

where  $\delta$  is the discount rate;  $C$  denotes the non-durable consumption;  $H$  denotes the housing stock<sup>3</sup>;  $\alpha^k$  is a vector of portfolio weights for financial assets and housing asset. The superscript  $k$  denotes the  $k^{th}$  individual, which I will omit for this and the next section to keep notation simple. I assume that the per period utility function satisfies the standard conditions, i.e. the utility function is concave, non-decreasing and twice differentiable in both arguments. the agents are heterogeneous in terms of their initial wealth. The utility function is non-separable in non-durable and housing consumption.

The price of the riskless bond  $B_t$  follows

$$dB_t = rB_t dt \quad (2)$$

The prices of the  $n$  risky assets are assumed to follow the stochastic differential equations:

$$dP_i = \mu_i P_i dt + \sigma_i P_i dz_i, i = 1, 2 \dots, n. \quad (3)$$

All  $z_i$ 's are potentially correlated one dimensional standard Brownian motions;  $\mu_i$ 's

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<sup>2</sup>I will relax this assumption by introducing adjustment costs in a later.

<sup>3</sup>I don't distinguish between housing stock and housing service here. An implicit assumption here is that housing service is proportional to housing stock.

the instantaneous expected returns and  $\sigma_i$  is the instantaneous volatility of  $i^{th}$  financial asset. Note that in this specification, I allow the expected returns to be time varying, although I suppress subscript  $t$  to simplify the notation.

The house price also follows a diffusion process:

$$dP_h = \mu_h P_h dt + \sigma_h P_h dz_h \quad (4)$$

where  $\mu_h$  and  $\sigma_h$  are instantaneous expected return and volatility of owner-occupied housing respectively.

The covariance matrix of these price processes is given by:

$$\Sigma = \begin{pmatrix} \sigma_1^2 & \cdots & \sigma_{1n} & \sigma_{1h} \\ \cdot & \cdots & \cdot & \cdot \\ \sigma_{n1} & \cdots & \sigma_{nn} & \sigma_{nh} \\ \sigma_{h1} & \cdots & \sigma_{hn} & \sigma_h^2 \end{pmatrix} \quad (5)$$

The Brownian motions are defined on the fixed probability space,  $\{\Omega, \mathcal{F}, P\}$ , with filtration  $\{\mathcal{F}_t\}$ , where  $\{\mathcal{F}_t\}$  is the filtration generated by all the Brownian motions and augmented by all  $P$ -null sets, i.e.  $\mathcal{F}$  and  $\mathcal{F}_t$  are both complete. Under this definition, prices, drift terms and the covariance matrix are all adapted to  $\{\mathcal{F}_t\}$ .

Denote individual wealth as  $W$ , and let  $\alpha_i, i = 1, 2, \dots, n$  denote the proportion of wealth invested in asset  $i$ , and let  $\alpha_h$  be the proportion invested in owner-occupied housing. The budget constraint, or the total wealth process follows:

$$dW = \left[ W \left( \sum_{i=1}^n [\alpha_i (\mu_i - r)] + \alpha_h (\mu_h - r) + r \right) - c \right] dt + W \left( \sum_{i=1}^n \alpha_i \sigma_i dz_i + \alpha_h \sigma_h dz_h \right) \quad (6)$$

## 2.2 Model Solution Without Adjustment Cost

First I consider the case where there are no adjustment costs for changing housing consumption, in this case, the state variables are the wealth level and house price, i.e. the value function has the following form:  $V = V(W, P_h)$ <sup>4</sup>. The housing price enters the value function because it is the quantity of housing that agents derive utility from but not the house value. Now assume that the value function is twice continuously differentiable in both arguments. I restrict the attention to all admissible controls, i.e. the controls that are adapted to the filtration  $\{\mathcal{F}_t\}$ , and under such controls, the differential equation of (6) is well defined, and has a unique solution. Under these assumptions, the solution of (6) is given by the following Hamilton-Jacobi-Bellman equation:

$$\begin{aligned} \delta V(W, P_h, t) = \max_{\{c, \alpha\}} \{ & U(C, H) + V_1 \left[ W \left( \sum_{i=1}^n [\alpha_i (\mu_i - r)] + \alpha_h (\mu_h - r) + r \right) - C \right] + \\ & V_2 \mu_h P_h + V_{12} W \left( \sum_{i=1}^n \alpha_i \sigma_{ih} P_h + \alpha_h \sigma_h^2 P_h \right) + \\ & \frac{1}{2} V_{11} W^2 \left( \sum_{i=1}^n \sum_{j=1}^n \alpha_i \alpha_j \sigma_{ij} + 2 \sum_{i=1}^n \alpha_i \alpha_h \sigma_{ih} + \alpha_h^2 \sigma_h^2 \right) + \frac{1}{2} V_{22} \sigma_h^2 P_h^2 \} \quad (7) \end{aligned}$$

where  $V_1, V_2$  are partial derivatives with respect to the first and second arguments of the value function, and  $V_{12}, V_{11}, V_{22}$  are second order partial derivatives of the value function.

The solution also satisfies the transversality condition:

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<sup>4</sup>Strictly speaking, the state variable should also include the time varying expected returns as they reflect stochastic investment opportunity set. I follow Merton (1980) and Jaganathan and Wang(1995) by assuming that the hedging demand for changing investment opportunity set is not sufficiently important. Thus I ignore these state variables from the onset. I essentially restrict the investors' decision rule not to be contingent on expected return with this assumption, or I assume implicitly that investors don't know the dynamics of expected returns. This assumption will not change most of the theoretical results, it just reduces some unknown pricing factors.

$$\lim_{T \rightarrow \infty} \delta^T V(W, P_h) = 0. \quad (8)$$

The following first order conditions are derived as the necessary condition for the HJB equation:

$$C : U_1(C, H) = V_1 \quad (9)$$

$$\alpha_i : V_1 W(\mu_i - r) + V_{12} W \sigma_{ih} P_h + V_{11} W^2 \left( \sum_{j=1}^n \alpha_j \sigma_{ij} + \alpha_h \sigma_{ih} \right) = 0, \text{ for } i = 1, 2, \dots, n \quad (10)$$

$$\alpha_h : V_1 W(\mu_h - r) + V_{12} W \sigma_h^2 P_h + V_{11} W^2 \left( \sum_{j=1}^n \alpha_j \sigma_{jh} + \alpha_h \sigma_h^2 \right) + U_2(C, H) W / P_h = 0 \quad (11)$$

From equation (9)-(11), I can solve for the consumption choice  $C$  and portfolio choice  $\alpha_i$ 's and  $\alpha_h$  as functions of the unknown value function. In particular, the portfolio choice can be solved explicitly as follows:

$$\alpha_i = \left[ \sum_{j=1}^n \xi_{ij} [(A(\mu_j - r) + B\sigma_{ih}P_h)] \right] + \xi_{ih} [(A(\mu_h - r) + B\sigma_{ih}P_h)] - \xi_{ih} \frac{U_2(C, H)/P_h}{V_{11}W} \quad (12)$$

$$\alpha_h = \left[ \sum_{j=1}^n \xi_{hj} [(A(\mu_j - r) + B\sigma_h^2P_h)] \right] + \xi_{hh} [(A(\mu_h - r) + B\sigma_h^2P_h)] - \xi_{hh} \frac{U_2(C, H)/P_h}{V_{11}W} \quad (13)$$

where

$$\Psi = \begin{pmatrix} \xi_{11} & \cdots & \xi_{1n} & \xi_{1h} \\ \cdot & \cdots & \cdot & \cdot \\ \xi_{n1} & \cdots & \xi_{nn} & \xi_{nh} \\ \xi_{h1} & \cdots & \xi_{hn} & \xi_{hh} \end{pmatrix} = \Sigma^{-1} \quad (14)$$

is the inverse of the covariance matrix.

$$A = -\frac{V_1}{WV_{11}} \text{ is the inverse of relative risk aversion.}$$

$$B = -\frac{V_{12}}{WV_{11}}$$

The relationship between  $\Psi$  and  $\Sigma$  helps to further simplify (13) and (14) to:

$$\alpha_i = \left[ \sum_{j=1}^n \xi_{ij} A(\mu_j - r) \right] + \xi_{ih} A(\mu_h - r) - \xi_{ih} \frac{U_2(C, H)/P_h}{V_{11}W} \text{ for } i = 1, 2, \dots, n \quad (15)$$

$$\alpha_h = \left[ \sum_{j=1}^n \xi_{hj} A(\mu_j - r) \right] + \xi_{hh} A(\mu_h - r) + BP_h - \xi_{hh} \frac{U_2(C, H)/P_h}{V_{11}W} \quad (16)$$

The demand for non-housing risky assets now has three components: the first component is the traditional demand for risky assets in the mean-variance framework, or it is the demand generated by agents who face a time-invariant investment opportunity set without owner-occupied housing; the second component is the demand when housing is taken as a risky asset; the third component is the demand for the asset as a vehicle to achieve intratemporal optimization. The third component of the demand for risky asset makes the model interesting, if there is no such demand for the purpose of intertemporal optimization, we can just take owner-occupied housing as another risky asset, then based on traditional CAPM argument, there should be a linear factor pricing model in which market portfolio return and real estate return are two common risk factor as in Kullmann (2003). Thus adding owner-occupied housing not only introduces real estate return as an additional common risk factor, it also adds something related to the demand for housing as a consumption good. The demand for owner-occupied housing has four components, while three of them have similar explanations as for risky asset, the fourth component, as

the third term in equation (16) is the demand for housing as intratemporal optimization of housing choice.

As a special case, if the covariance matrix is block diagonal, i.e.  $\sigma_{ih}$ 's are all 0, it is easy to see that the second and third term of the demand for risky asset disappear, which makes the model go back to the traditional CAPM case as in Flavin and Nakagawa (2004)<sup>5</sup>. But this is not true in general. The following proposition summarizes this finding.

**Proposition 1 ((Failure of Two Fund Separation and Three Fund Separation))**

*Neither two fund separation nor three fund separation holds under general conditions, the market portfolio is not mean variance efficient and traditional CAPM fails as a consequence.*

This can be seen because the ratio between any two risk assets demand not only depends on the characteristics of the risky assets, but also depends on the characteristics of the investor, as reflected by  $A$ ,  $B$ , and  $\frac{U_2(C,H)/P_h}{V_{11}W}$ . Notice also that  $A$ ,  $B$ , and  $\frac{U_2(C,H)/P_h}{V_{11}W}$  not only depend on investors preference, i.e. form of the utility function, but also clearly depend on investor's wealth level. This means that under general condition, aggregation of the demand for risky assets depends on the distribution of the wealth levels across investors even when the investors have identical utility functions.

The failure of two-fund separation is caused by the fact that owner-occupied housing is also a consumption good. The asset demand caused by intratemporal choice of non-durable consumption and housing makes the asset demand heterogeneous because of the heterogeneity of households.

However there are two special cases, under which two fund or three fund separation does hold. The following corollary states the specific condition.

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<sup>5</sup>Although they reach the conclusion in the case of adjustment costs

**Corollary 2** (*Conditions for Three Fund Separation*) *If the covariance matrix is block diagonal, then three fund separation holds. The three separating funds are the risk-free asset, the market portfolio of non-housing risky assets and housing. In this case the market portfolio of non-housing risky asset is mean-variance efficient and the traditional CAPM holds.*

This block diagonal condition given above is the driving force for the results in Grossman and Laroque (1990) and Flavin and Nakagawa (2004). In Grossman and Laroque (1990), although they do not explicitly assume the covariance matrix is block diagonal, house price in their model is deterministic, which leads to no correlation between house price and any stock price.

**Corollary 3** (*Conditions for Two Fund Separation*) *If the individual preference satisfies that  $\frac{U_2(C,H)/P_h}{AV_{11}W}$  is constant for all investors, then two fund separation holds, the two separating funds are the market portfolio (including both non-housing risky assets and housing) and the risk-free asset. As a consequence, the traditional CAPM holds with the market portfolio of non-housing risky assets replaced by the market portfolio of both non-housing risky assets and housing.*

Since conditions for both corollary are difficult to satisfy, the asset pricing implications in the following sections concern the very general case in which both two-fund and three-fund separation fails.

## 2.3 Equilibrium Asset Pricing without Adjustment Cost

Let  $M$  be the market portfolio of total risky assets, non-housing and housing, and let  $\omega_i$ 's and  $\omega_h$  be the market portfolio weights. By definition,

$$M = \sum_{k=1}^K \left[ \begin{array}{l} \sum_{i=1}^n \left( A^k W^k \left[ \sum_{j=1}^n \xi_{ij}(\mu_j - r) + \xi_{ih}(\mu_h - r) \right] - \xi_{ih} \frac{U_2^k(C^k, H^k)/P_h}{V_{11}^k} \right) + \\ A^k W^k \left[ \sum_{j=1}^n \xi_{hj}(\mu_j - r) \right] + \xi_{hh}(\mu_h - r) + B P_h W^k - \xi_{hh} \frac{U_2^k(C^k, H^k)/P_h}{V_{11}^k} \end{array} \right]$$

Also by definition, the following equations hold for individual assets:

$$\omega_i M = \sum_{k=1}^K \left\{ A^k W^k \left[ \sum_{j=1}^n \xi_{ij}(\mu_j - r) + \xi_{ih}(\mu_h - r) \right] - \xi_{ih} \frac{U_2^k(C^k, H^k)/P_h}{V_{11}^k W^k} \right\} \text{ for } i = 1, 2, \dots, n \quad (17)$$

$$\omega_h M = \sum_{k=1}^K \left\{ A^k W^k \left[ \sum_{j=1}^n \xi_{hj}(\mu_j - r) + \xi_{hh}(\mu_h - r) \right] + B^k P_h - \xi_{hh} \frac{U_2^k(C^k, H^k)/P_h}{V_{11}^k W^k} \right\} \quad (18)$$

where superscript  $k$  denotes  $k$ th individual.

Now redefine:  $A = \sum_{k=1}^K A^k W^k$  and  $B = \sum_{k=1}^K B^k$  with a little abuse of notation and solve

equation (17) and (18) for the expected excess returns,

$$\mu_i - r = \frac{M}{A} \left( \sum_{j=1}^n \sigma_{ij} \omega_j + \sigma_{ih} \omega_h \right) + \frac{B}{A} P_h \sigma_{ih} \quad \text{for } i = 1, 2, \dots, n \quad (19)$$

and

$$\mu_h - r = \frac{M}{A} \left( \sum_{j=1}^n \sigma_{jh} \omega_j + \sigma_{hh} \omega_h \right) + \frac{B}{A} P_h \sigma_h^2 + \frac{\sum_{k=1}^K \frac{U_2^k(C^k, H^k)}{V_{11}^k}}{A P_h} \quad (20)$$

Notice that equation (19) can be seen as a two factor linear asset pricing model, because the formula in the bracket is just the correlation of asset  $i$  with the market

portfolio (including housing), thus market portfolio is one risk factor, the other risk factor is the housing return, which is the second term in the right hand side of equation (19). i.e.  $\sum_{j=1}^n \sigma_{ij}\omega_j + \sigma_{ih}\omega_h = \sigma_{iM}$ , where  $\sigma_{iM} \equiv cov(dP_i/P_i, dM/M)$ . The housing return in equation (20), however is, a little more complicated, since it also reflects consumption role of housing, as in the third term of the right hand side of equation (20).

According to equation (19) and (20), the market portfolio return satisfies:

$$\mu_{M-r} = \frac{M}{A} \left( \sum_{j=1}^n \sigma_{jM}\omega_j + \sigma_{hM}\omega_h \right) + \frac{B}{A} P_h \sigma_h^2 + \frac{\omega_h \sum_{k=1}^K \frac{U_2^k(C^k, H^k)}{V_{11}^k}}{AP_h}, \dots, n \quad (21)$$

According to (20) (21), I can rewrite (19) as:

$$\mu_i - r = \beta_{im} \left[ (\mu_M - r) - \frac{\omega_h \sum_{k=1}^K \frac{U_2^k(C^k, H^k)}{V_{11}^k}}{AP_h} \right] + \beta_{ih} \left[ (\mu_h - r) - \frac{\sum_{k=1}^K \frac{U_2^k(C^k, H^k)}{V_{11}^k}}{AP_h} \right] \quad (22)$$

where  $\beta_{im}$  and  $\beta_{ih}$  are multivariate regression coefficients of return of asset  $i$  on market return and housing return respectively, i.e. they are multivariate  $\beta'$ s.

To get more concrete expression for the third term, I adopt the following functional form of the utility function.

$$U(C, H) = \frac{(C^\eta H^{1-\eta})^{1-\gamma}}{1-\gamma} \quad (23)$$

Under this Cobb-Douglas specification of utility function, the intratemporal elasticity of substitution between non-durable consumption and housing is 1, which is not a bad approximation of the actual elasticity.<sup>6</sup>

<sup>6</sup>Piazzesi et al (2005) show that the elasticity of substitution is only slightly bigger than 1.

Under this specific assumption, the utility function is homogeneous of degree  $(1 - \gamma)$  in  $C$  and  $H$ . This, in combination with linearity of the budget constraint (5) implies that the value function is also homogeneous of degree  $(1 - \gamma)$  in wealth  $W$ . With this properties of the value function and utility function,

$$\frac{\sum_{k=1}^K \frac{U_2^k(C^k, H^k)}{V_{11}^k}}{AP_h} = - \frac{(1 - \alpha) \sum_{k=1}^K W^k \frac{C^k}{H^k}}{\alpha P_h W} \quad (24)$$

I can further rewrite equation (22) as:

$$\mu_{i-r} = \beta_{im} \left[ (\mu_{M-r}) + \frac{\omega_h (1 - \eta) \sum_{k=1}^K W^k \frac{C^k}{H^k}}{\eta P_h W} \right] + \beta_{ih} \left[ (\mu_h - r) + \frac{(1 - \eta) \sum_{k=1}^K W^k \frac{C^k}{P_h H^k}}{\eta W} \right] \quad (25)$$

Notice that  $\frac{C^k}{H^k P_h}$  is the ratio of expenditure on non-durable consumption to the housing value, which I call  $ch$

$\frac{\sum_{k=1}^K W^k \frac{C^k}{H^k P_h}}{K}$  is an aggregation across all investors, which cannot be measured explicitly in practice. In the data, we observe that the consumption-to-housing ratio is very stable over time and across individual investors, thus we can approximate it by an aggregate measure  $\frac{C}{HP_h}$ , where  $C$  is the aggregate non-durable consumption, and  $H$  is the aggregate housing stock. Notice also that  $\omega_h = \frac{HP_h}{W}$ . I obtain the following asset pricing equation which I can estimate empirically:

$$\mu_{i-r} \approx \beta_{im} \left[ (\mu_{M-r}) + \frac{(1 - \eta) C}{\eta W} \right] + \beta_{ih} \left[ (\mu_h - r) + \frac{(1 - \eta) C}{\eta HP_h} \right] \quad (26)$$

The implication of equation (26) is that, in time series,  $ch$ , as well the consumption-

to-wealth ratio,  $cay$ , can predict the expected returns of financial assets. and moreover, they can both be used as conditioning variable in cross-sectional test of asset pricing models. Thus this theoretical model can rationalize the empirical findings of Lettau and Ludvigson (2001a, 2001b), in which they show that  $cay$  can predict expected returns. Notice that since asset returns are time varying, 26 holds in a conditional sense, but for the purpose of empirical testing, I need a unconditional version of 26. To do that, I use the following proposition.

**Proposition 4** *The stochastic discount factor of this economy can be return as:*

$$M_{t+1} = a_t + b_t R_{M_{t+1}} + c_t R_{ht+1} \quad (27)$$

where  $a_t$ ,  $b_t$  and  $c_t$  are linear functions of  $\frac{C}{W}$  and  $\frac{C}{HP_h}$ . i.e.

$$\begin{aligned} a_t &= a_1 + a_2 \frac{C_t}{W_t} + a_3 \frac{C_t}{H_t P_{ht}} \\ b_t &= b_1 + b_2 \frac{C_t}{W_t} + b_3 \frac{C_t}{H_t P_{ht}} \\ c_t &= c_1 + c_2 \frac{C_t}{W_t} + c_3 \frac{C_t}{H_t P_{ht}} \end{aligned}$$

Based on the proposition above, the model can be tested unconditionally in the same way as in Lettau & Ludvigson (2001b). Notice that in Lettau and Ludvigson assume that their  $cay$  enters the stochastic discount factor linearly, which is crucial in carrying out their cross-sectional asset pricing test. Here the linearity emerges from my model. The fact that both  $ch$  and  $cay$  enter the stochastic discount factor linearly will enable me to test the model using the Fama-MacBeth procedure in the empirical part of this paper.

## 2.4 Asset Pricing With Adjustment Costs

Now I consider the case where there are substantial adjustment costs associated with change of housing consumption level, in this case, the quantity of housing consumption is also a state variable, because it affects the policy of optimal adjustment. Because I focus on the asset pricing implication of the model, I do not need to solve the full model. Since there are substantial adjustment costs, the adjustment is infrequent, and occurs with probability zero. In the no adjustment region, which occurs with probability 1, the Hamilton-Jacobi-Bellman Equation is given by:

$$\begin{aligned} \delta V(W, P_h, H, t) = & \max_{\{c, \alpha\}} \{U(C, H) + V_1 \left[ W \left( \sum_{i=1}^n [\alpha_i (\mu_i - r)] + \alpha_h (\mu_h - r) + r \right) - C \right] + \\ & V_2 \mu_h P_h + V_{12} W \left( \sum_{i=1}^n \alpha_i \sigma_{ih} P_h + \alpha_h \sigma_h^2 P_h \right) + \\ & \frac{1}{2} V_{11} W^2 \left( \sum_{i=1}^n \sum_{j=1}^n \alpha_i \alpha_j \sigma_{ij} + 2 \sum_{i=1}^n \alpha_i \alpha_h \sigma_{ih} + \alpha_h^2 \sigma_h^2 \right) + \frac{1}{2} V_{22} \sigma_h^2 P_h^2 \} \quad (28) \end{aligned}$$

It looks almost the same as (7), the only difference is that in this problem the housing consumption is not a choice variable, rather, the agents choose nondurable consumption and financial asset holdings taking housing as given.

The first order necessary conditions are given as:

$$C : U_1(C, H) = V_1 \quad (29)$$

$$\alpha_i : V_1 W (\mu_i - r) + V_{12} W \sigma_{ih} P_h + V_{11} W^2 \left( \sum_{j=1}^n \alpha_j \sigma_{ij} + \alpha_h \sigma_{ih} \right) = 0, \text{ for } i = 1, 2, \dots, n \quad (30)$$

Solve for the portfolio weights from equation (30) as before

$$\alpha_i W = \sum_{j=1}^n \xi_{ij} [A(\mu_j - r) + B P_h \sigma_{jh} - \alpha_h \sigma_{jh} W] \quad \text{for } i = 1, 2, \dots, n \quad (31)$$

Again the demand for financial assets has three parts that are easy to interpret, the first part is the traditional demand for diversification of financial assets, the second part is generated by adjustment to smooth consumption when there is housing, the third part is the demand to hedge house price fluctuation.

In this case, the two fund separation does not hold either if the correlations between the risky assets and housing are not all zero.

Now I consider the equilibrium asset pricing equilibrium with adjustment cost. Following the same argument in last section, the asset market equilibrium implies that:

$$\omega_i M = \sum_{k=1}^K \left\{ A^k \left[ \sum_{j=1}^n \xi_{ij} (\mu_j - r) \right] + B^k \left[ \sum_{j=1}^n \xi_{ij} \sigma_{jh} P_h \right] - \sum_{j=1}^n W \xi_{ij} \sigma_{jh} \alpha_h \right\} \text{ for } i = 1, 2, \dots, n \quad (32)$$

The asset return can then be solved as:

$$\mu_i - r = \frac{M}{A} \sigma_{iM} + \frac{B}{A} P_h \sigma_{ih} - \frac{1}{A} \alpha_h \sigma_{ih}, \text{ for } i = 1, 2, \dots, n \quad (33)$$

Aggregate over all the financial asset, the market return is thus given by:

$$\mu_M - r = \frac{M}{A} \sigma_M^2 + \frac{B}{A} P_h \sigma_{hM} - \frac{1}{A} \alpha_h \sigma_{hM} \quad (34)$$

Combining equation (33) and (34), the final linear pricing equation is:

$$\mu_i - r = \frac{\sigma_{iM}}{\sigma_M^2} \left[ (\mu_M - r) - \frac{B}{A} P_h \sigma_{hM} + \frac{1}{A} \alpha_h \sigma_{hM} \right] + \sigma_{ih} \left( \frac{B}{A} P_h - \frac{\alpha_h}{A} \right), \text{ for } i = 1, 2, \dots, n \quad (35)$$

This is again a two factor pricing model, in which the market portfolio and real estate return are two pricing factors, but the cross-sectional coefficient are no longer risk premiums, and are also different from the term in last section, although the qualitative feature

of the cross-sectional pricing remain unchanged. Different from the last section also, since there is no first-order condition for housing, there is no simplification of (35) similar to (25) that leads to simple aggregate quantities that can be measured explicitly. Thus for empirical testing, I will focus on the implication of the no adjustment costs case, for it is easier to implement.

### 3 Data and Empirical Strategy

The empirical testing of the model is divided into two parts. The first part devotes to the time series implications of the model, since the model implies that the consumption to housing ratio,  $ch$  as well as the consumption-to-wealth ratio  $cay$ , predicts asset returns, I test the empirically a model of the following form:

$$r_{t+k} = \alpha + \beta_1 ch_t + \beta_2 cay_t + \varepsilon \tag{36}$$

for different predicting horizon  $k$ . I do this for different measures of asset returns, first the value weighted market returns, then the risk premium of value weighted market portfolio.

The non-durable consumption and housing consumption are from NIPA tables. But I use a different definition of nondurable consumption than the NIPA table. The non-durable consumption in this paper is equal to the non-durable consumption in NIPA table, plus service in NIPA table, minus shoes and clothes consumption, minus housing service. Because the imputed housing service in NIPA table is actually from the durable consumption goods, owner-occupied housing. This is not part of the non-durable consumption in the model above. The data is quarterly from 1952Q1-2005Q4. The value of housing stock is from Flow of Funds table. The consumption to housing ratio  $ch$  is defined as the ratio of nondurable consumption to the value of total housing stock. The

non-durable consumption to wealth ratio is from Martin Lettau’s website, although their definition of non-durable consumption is a little different from mine (they include housing service), I still use their *cay* for the purpose of comparing my results with theirs. The value weighted market return , also quarterly from 1952Q1-2005Q4 is from CRSP, the T-bill rate is from Kenneth French’s website.

In the second part of empirical tests, I study the cross-sectional implications of the model. First the model implies a two factor pricing model, in which market portfolio return and real estate return are two common risk factor; second, the model also implies that the consumption to housing ratio, *ch* as well as *cay* can be used as conditioning variable in cross-sectional asset return test. In the cross-sectional test, the asset returns are 25 size and book-to-market portfolios from Kenneth French’s website., the real estate return is computed from OFHEO housing price index, consumption measures are from NIPA tables. I use Fama MacBeth regression to implement the cross-sectional tests, because as mentioned in Lettau and Ludvigson, Fama-MacBeth regression has important advantage over other methods in the case when there are only moderate numbers of time series observations, which is exactly the situation here, since the real estate return data is available only for a short time period (1967-2005). I will compare the empirical performance of several asset pricing model, either unconditionally or conditionally, where the conditioning variable is *ch*. For the general setup,if the risk factor in a conditional model is  $f_t$ , the unconditional test is:

$$E[R_{it+1}] = E[R_{0t}] + \beta' \lambda \tag{37}$$

where  $\beta = cov(f, f)^{-1}cov(R_i, f)$ .If the model is tested conditional on the conditional variable  $z$ , then the factor space can be augmented as  $f'_t = [f_t \ z \ f_t z]$ , and  $\beta$  becomes  $\beta = cov(f', f')^{-1}cov(R_i, f')$ , and the above equation holds cross-sectionally.

## 4 Empirical Results

### 4.1 Predictability Regression Results

I first show the predictability results using  $ch$  and  $cay$  as predicting variables. Table-1 shows the OLS estimates of market return predictability. The first panel shows the estimates using  $cay$  alone as the predicting variable; the second panel shows the estimates using  $ch$  alone and the last panel reports the estimates using both  $cay$  and  $ch$ . The forecast horizon ranges from 1 quarter to 8 quarters. In each panel, the first row reports the slope coefficients estimates of the predictive regressions for different forecasting horizons; the second row reports the standard errors of the coefficient estimates, which are calculated with Newey-West autocorrelation and heteroskedasticity consistent estimates, as is now standard in the literature; the third row of each panel reports the adjusted R-square (in percentage points) of the predictive regressions. The first panel shows that  $cay$  predicts 5.34% of next quarterly return, and  $R^2$  increases to 22.46% at two year horizon. Moreover all the slope coefficient estimates are statistically significant at 5% level. The second panel shows that  $ch$  also predicts market return, the slope coefficient estimates are significant and  $R^2$  is comparable to that of using  $cay$ . Comparing the two, we see that  $ch$  performs slightly better in short horizon, while  $cay$  is better in longer horizon. The last panel shows most slope coefficient estimates are insignificant, although the  $R^2$  is still large. This is because that the correlation between  $cay$  and  $ch$  is very high, about 0.9, and this makes the standard error very big. From the last panel we also see that  $ch$  performs better in short horizon and worse in longer horizon. Comparing the adjusted  $R^2$  in last panel with that in the first and second panel, I see that there is not much gains in terms of  $R^2$  for including the other predicting variable, this is again caused by the high correlation between  $cay$  and  $ch$ . Table-2 shows the OLS regression estimates for excess returns, the structure of the table is identical to table-1. The pattern of the estimates are very similar to the estimates for market returns, only that the  $R^2$  is slightly lower.

The high correlation between *cay* and *ch* makes it difficult to separate their effects in a predictive regression with both as predicting variables. While in this paper, I want to see whether *ch* adds something new in predicting market returns. To do that, I first regress *ch* on *cay*, and get the residuals from the regression, then I run a predictive regression of returns on *cay* and the residuals. The results are reported in table-3. Panel A reports the results for market returns; panel B reports the results for excess returns. Not surprisingly, coefficient estimates on *cay* are statistically significant for all horizons; and the estimates on the residual are significant at the one quarter horizon. Moreover, the  $R^2$  is also higher at the one quarter horizon, as compared to that from the regression with *cay* alone. These estimates shows that although *ch* is highly correlated with *cay*, it plays its own role in predicting asset returns, especially at short horizon.

Since what matters in conditional cross sectional tests is just one quarter ahead predictability, I will only use *ch* in testing conditional models for too reason: first *ch* is shown to perform slightly better than *cay* in predicting one quarter ahead market returns; second, including one more conditional variable will introduce three additional factors in testing a conditional two factor model, which will reduce the power of test substantially given that I only test 25 portfolios.

## 4.2 Cross-Section Test Results

Using Fama-French 25 size and BM sorted portfolios, I study the empirical performance of various beta representations of asset pricing models. Table-4 is the tabulation of the Fama-MacBeth regression results of these models. It reports the estimates of  $\lambda$  coefficients, the Shanken-corrected  $t$ -statistics for these coefficients, and the adjusted  $R^2$ .

To compare the performance of various models, I begin with familiar unconditional models.

### (1) Traditional CAPM

In traditional CAPM, the only pricing factor is the market return, here I use the CRSP value-weighted return as a proxy for the unobservable market return, as is standard in literature. The cross-sectional implication of CAPM has the following form:

$$E[R_{it+1}] = \alpha + \beta_{im}\lambda_m \quad (38)$$

The first row of table-2 reports the results of traditional CAPM. The t-statistics for  $\lambda_m$  shows that the beta on the value weighted return is not a statistically significant determinant of the cross-section of average returns. Moreover, the  $R^2$  is low, only 2% variation of the expected return of the 25 portfolios can be explained by CAPM. These results are now familiar.

### (2) Unconditional Two Factor Model including Housing

Now I study the performance of the two factor model derived in the above theoretical part, in which the market portfolio return and real estate return are two pricing factors. The real estate return is approximated by the OFHEO price index return, the market return is again the value weighted CRSP return. The cross-sectional implication of the model is:

$$E[R_{it+1}] = \alpha + \beta_{im}\lambda_m + \beta_{iH}\lambda_H \quad (39)$$

The empirical results in the second row show that the coefficient estimates of  $\lambda_m$  is negative and insignificant, while  $\lambda_H$  of real estate return is positive and significant, the  $R^2$  increases significantly relative that of traditional CAPM. The results here are similar to the results obtained in Kullmann (2003).

### (3) Consumption Based CAPM

In a consumption-based CAPM, the consumption growth rate is the single pricing

factor, i.e. the following equation describes the cross-sectional determination of expected returns:

$$E[R_{it+1}] = \alpha + \beta_{ic}\lambda_c \quad (40)$$

$\beta_{ic}$  is the time series regression of asset return on the non-durable consumption growth rate. The third row of table-2 reports the results of consumption based CAPM. The estimates of the cross-sectional coefficient  $\lambda_c$  is positive but insignificant, the adjusted  $R^2$  is about 0.2, slightly higher than the traditional CAPM, but lower than the two factor model above.

#### (4) Fama-French 3-Factor Model

Row 4 reports the results for Fama-French three-factor model, which has the following form:

$$E[R_{it+1}] = \alpha + \beta_{im}\lambda_m + \beta_{iHML}\lambda_{HML} + \beta_{iSMB}\lambda_{SMB} \quad (41)$$

the SMB and HML factors are again from French's website. Compared with the traditional CAPM, the three-factor model is a great success since it can explain almost 70% of the variation in cross-sectional expected returns. In this model, the  $t$ -statistics of market return factor and SMB factor are statistically significant, especially the SMB factor, which is highly significant. The three-factor model has been thought as the most successful empirical asset pricing model.

Now I move on to the conditional pricing model by using the constructed  $ch$  as the conditioning variable. The rationale of using  $ch$  as the conditioning variable follows Lettau and Ludvigson (2001b).

#### (5) Conditional CAPM

The conditional CAPM takes the form:

$$E[R_{it+1}] = \alpha + \beta_{iz}\lambda_z + \beta_{im}\lambda_m + \beta_{imz}\lambda_{mz} \quad (42)$$

$z$  denotes the conditioning variable,  $ch$ , in this paper. The empirical results of conditional CAPM are reported in the fifth row of table-2. The results show that the coefficient estimate on the conditioning variable itself is not statistically significant, while the estimates on market return and conditional market return are significant, although the estimates on market return is negative, which is not unusual. The adjusted  $R^2$  improves significantly over the unconditional CAPM, the conditional CAPM can explain almost 55% of the cross-sectional expected returns.

#### (6) Conditional Consumption-Based CAPM

Now I use  $ch$  to study the conditional consumption-based CAPM, the conditional model is given as:

$$E[R_{it+1}] = \alpha + \beta_{iz}\lambda_z + \beta_{ic}\lambda_c + \beta_{icz}\lambda_{cz} \quad (43)$$

Though the empirical model does not produce sensible significant coefficient estimates (the negative estimates on consumption growth rate is contrary to the economic theory), the  $R^2$  improves over the unconditional consumption-based CAPM.

#### (7) Conditional Two Factor Model with Real Estate

Now I study the conditional model performance of two-factor pricing model derived in the theoretical part, i.e.:

$$E[R_{it+1}] = \alpha + \beta_{iz}\lambda_z + \beta_{im}\lambda_m + \beta_{imz}\lambda_{mz} + \beta_{iH}\lambda_H + \beta_{iHz}\lambda_{Hz} \quad (44)$$

The empirical power of this conditional two-factor model is comparable with Fama-French 3-factor model. The model explains 65% of the expected return variations. While the point estimates of the cross-sectional regression coefficient on market return, the conditional market return and conditional real estate return are statistically significant,

and the coefficient of the conditioning variable itself is not significant, as in the 7<sup>th</sup> row of table-2.

Figure-1 plots the model fitted return *vs* realized return for various models discussed above, the up-left figure plots the fitted return for traditional CAPM, as shown in the figure, CAPM explains visually none of the variation in average returns on these portfolios, while the model performance of unconditional two factor model is much better than traditional CAPM, as shown in the up-right figure. The poor performance of consumption-based CAPM is also obvious from the middle-left figure. The best performance, as visually from figure-2 is the Fama-French three-factor model, while the conditional two-factor model performs almost the same as the three-factor model. Moreover, conditional models conditioning on *ch* perform much better than their unconditional counterparts.

## 5 Conclusion

Owner-occupied housing plays dual role in household decision, on the one hand it is an important consumption good, on the other hand, it is dominant asset in most household portfolio. This paper studies the implication of this special feature of housing on asset pricing in a simple continuous time framework. The theoretical results show that the with owner-occupied housing, the market portfolio is not mean-variance efficient, and traditional CAPM fails; however, a conditional linear factor pricing model can still be derived, in which the market portfolio return and housing return are two pricing factors. The nondurable consumption to housing ratio, *ch*, which enters linearly the stochastic discount factor, can predict the expected return of financial assets, thus can also be used as conditioning variable in cross-sectional asset pricing models. As a by product, this model also rationalizes the use of *cay* in Lettau and Ludvigson (2001a, 2001b) as predicting variable in predictive regression and conditioning variable in conditional asset pricing tests.

The empirical predictability test shows that  $ch$  can predict market return, as well as market risk premium, at different horizons. The regression coefficient estimates are not only statistically significant but also economically large, as one standard deviation change of  $ch$  leads to almost 2.5% change in expected return. These results thus provide support to the theoretical asset pricing model of this paper.

The cross-sectional asset pricing test show that the unconditional two factor model derived theoretically performs much better than the traditional CAPM. The conditional models conditioning on  $ch$  have much higher explanatory power of cross-sectional expected returns than the unconditional versions. Further the conditional two factor model perform almost as well as Fama-French three-factor model. This shows that the theoretically grounded factor model is a good description of the real economy.

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## 7 Appendix

A. Derivation of Equation 22

Multiplying both sides of equation (19) by  $\omega_i$ , I get

$$\omega_i(\mu_i - r) = \frac{M}{A}\omega_i \left( \sum_{j=1}^n \sigma_{ij}\omega_j + \sigma_{ih}\omega_h \right) + \frac{B}{A}P_h\sigma_{ih}\omega_i \quad \text{for } i = 1, 2, \dots, n$$

Multiplying both sides of equation (19) by  $\omega_h$ , I get

$$\omega_h(\mu_h - r) = \frac{M}{A}\omega_h \left( \sum_{j=1}^n \sigma_{jh}\omega_j + \sigma_{ih}\omega_h \right) + \frac{B}{A}\omega_h P_h \sigma_h^2 + \frac{\omega_h \sum_{k=1}^K \frac{U_2^k(C^k, H^k)}{V_{11}^k}}{AP_h}$$

Sum over the  $n+1$  equations, I get

$$\mu_M - r = \frac{M}{A}\sigma_M^2 + \frac{B}{A}P_h\sigma_{hM} + \frac{\omega_h \sum_{k=1}^K \frac{U_2^k(C^k, H^k)}{V_{11}^k}}{AP_h} \quad (45)$$

Where  $\sigma_M^2 = \sum_{i=1}^n \omega_i \left( \sum_{j=1}^n \sigma_{jh}\omega_j + \sigma_{ih}\omega_h \right)$  is volatility for market return.

Given equation 45 and equation (20), I can solve for  $\frac{M}{A}$  and  $\frac{B}{A}P_h$  as:

$$\frac{M}{A} = \frac{\sigma_h^2(\mu_M - r) + (\omega_h\sigma_h^2 - \sigma_{hm}) \sum_{k=1}^K \frac{U_2^k(C^k, H^k)}{WV_{11}^k} - \sigma_{hm}(\mu_h - r)}{\sigma_M^2\sigma_h^2 - \sigma_{hM}^2} \quad (46)$$

and

$$\frac{B}{A}P_h = \frac{(\sigma_{hM} - \sigma_M^2)(\mu_M - r) + (\omega_h\sigma_{hm} - \sigma_M^2) \sum_{k=1}^K \frac{U_2^k(C^k, H^k)}{WV_{11}^k}}{\sigma_{hM}^2 - \sigma_M^2\sigma_h^2} AP_h \quad (47)$$

Substitute 46 and 47 into equation (19) and rearrange, I get equation 22. where

$$\beta_{iM} = \frac{\sigma_{iM}\sigma_h^2 - \sigma_{ih}\sigma_{hM}}{\sigma_M^2\sigma_h^2 - \sigma_{hM}^2}$$

and

$$\beta_{ih} = \frac{\sigma_{ih}\sigma_M^2 - \sigma_{iM}\sigma_{hM}}{\sigma_M^2\sigma_h^2 - \sigma_{hM}^2}$$

are the multivariate beta's.

## B. Derivation of Equation 24

By homotheticity, the value function is homogeneous of degree  $1 - \gamma$  in  $W$ , i.e. the value function can be written as:  $V(W, P_h, t) = f(t, P_h) \frac{W^{1-\gamma}}{1-\gamma}$  for some function  $f(t, P_h)$ . Then the first order partial derivative with respect to  $W$  is

$$V_1 = f(t, P_h)W^{-\gamma}$$

and the second order partial derivative with respect to  $W$  is

$$V_{11} = -\gamma f(t, P_h)W^{-\gamma-1}$$

Thus

$$A = \frac{W}{\gamma}$$

The first order condition in equation (9) then can be written as:

$$f(t, P_h)W^{-\gamma} = (C^\alpha H^{1-\alpha})^{-\gamma} \alpha C^{\alpha-1} H^{1-\alpha}$$

Thus

$$U_2(C, H) = (C^\alpha H^{1-\alpha})^{-\gamma} (1-\alpha) C^\alpha H^{-\alpha} = \frac{(1-\alpha)}{\alpha} f(t, P_h) W^{-\gamma} \frac{C}{H}$$

Thus

$$\frac{\sum_{k=1}^K \frac{U_2^k(C^k, H^k)}{V_{11}^k}}{AP_h} = - \frac{\frac{(1-\alpha)}{\alpha} \sum_{k=1}^K W^k \frac{C^k}{H^k}}{P_h W}$$

### C. Proof of Proposition 4

Rewrite the stochastic discount factor as follows:

$$m_{t+1} = A + B'f \tag{48}$$

where  $A = a_t + b_t \mu_M + c_t \mu_h$ ,  $B = (b_t, c_t)'$ ,  $f = (r_M - \mu_M, r_h - \mu_h)'$

I have:

$$E_t(m_{t+1} r_i) = 1$$

thus

$$\begin{aligned} E_t(r_i) &= \frac{1}{E_t(m_{t+1})} - \frac{E_t(r_i f') B}{E_t(m_{t+1})} \\ &= \frac{1}{E_t(m_{t+1})} - \frac{E_t(r_i f') E_t(ff')^{-1} E_t(ff') B}{E_t(m_{t+1})} \\ &= r - \beta_i \frac{E_t(ff') B}{r} \end{aligned}$$

where  $\beta_i = E_t(r_i f') E_t(ff')^{-1}$  is the multivariate beta.

Substitute back  $A, B$  and  $f$ , and compare the above equation with equation 26, I get

$$\begin{pmatrix} b_t \\ c_t \end{pmatrix} = r \begin{pmatrix} \sigma_M^2 & \sigma_{hM} \\ \sigma_{hM} & \sigma_h^2 \end{pmatrix}^{-1} \begin{pmatrix} (\mu_M - r) + \frac{(1 - \alpha)C}{W} \\ (\mu_h - r) + \frac{(1 - \alpha)C}{\alpha HP_h} \end{pmatrix} \quad (49)$$

which clear indicates that  $b_t, c_t$  are linear functions of  $\frac{C}{W}$  and  $\frac{C}{HP_h}$ , while  $a_t = 1/r$  is a trivial linear function of  $\frac{C}{W}$  and  $\frac{C}{HP_h}$

■

Table-1 Predictability of Market Return

This table reports estimates from OLS regression of market excess return on lagged *cay*, *ch* separately and combined. The market return here is the value weighted quarterly market returns from CRSP. *cay* is the ratio of consumption to wealth from Martin Lettau’s website, *ch* is the ratio of nondurable consumption to housing for homeowners. Column 1-8 report the estimates for forecasting horizons ranging from 1 to 8 quarters. The Newey-West corrected standard errors appear in parentheses below the coefficient estimates. Significant estimates at 5% level are highlighted in bold face. Regressions use data from the fourth quarter of 1952 to the fourth quarter of 2005.

Horizon(k)		1	2	3	4	5	6	7	8
<i>cay</i>	<i>cay</i>	<b>155.07</b>	<b>289.84</b>	<b>391.09</b>	<b>506.3</b>	<b>606.22</b>	<b>711.53</b>	<b>790.67</b>	<b>861.81</b>
		(45.95)	(82.72)	(112.9)	(142.7)	(160.11)	(101.0)	(193.5)	(220.9)
	$R^2$	5.34	8.50	10.62	13.72	15.95	18.80	20.44	20.44
<i>ch</i>	<i>ch</i>	<b>35.16</b>	<b>57.26</b>	<b>74.16</b>	<b>93.02</b>	<b>115.4</b>	<b>128.8</b>	<b>148.17</b>	<b>150.31</b>
		(9.16)	(17.05)	(20.46)	(25.06)	(18.83)	(20.24)	(36.63)	(36.09)
	$R^2$	6.98	8.47	9.74	11.91	14.76	15.93	18.49	17.60
<i>cay</i> and <i>ch</i>	<i>cay</i>	-3.37	156.16	268.5	396.89	406.83	<b>594.79</b>	<b>562.83</b>	<b>818.37</b>
		(86.73)	(116.5)	(179.2)	(237.3)	(239.3)	(227.7)	(234.4)	(278.4)
	<i>ch</i>	<b>35.74</b>	30.14	27.63	24.52	45.04	26.23	51.25	9.76
		(17.49)	(25.6)	(30.61)	(38.43)	(39.37)	(36.79)	(39.39)	(41.15)
	$R^2$	6.98	8.64	10.52	13.51	16.10	18.57	20.59	22.10

Table-2 Predictability of Market Excess Return

This table reports estimates from OLS regression of market excess return on lagged *cay*, *ch* separately and combined. The market return here is the value weighted quarterly market returns from CRSP. *cay* is the ratio of consumption to wealth from Martin Lettau's website, *ch* is the ratio of nondurable consumption to housing for homeowners. Column 1-8 report the estimates for forecasting horizons ranging from 1 to 8 quarters. The Newey-West corrected standard errors appear in parentheses below the coefficient estimates. Significant estimates at 5% level are highlighted in bold face. Regressions use data from the fourth quarter of 1952 to the fourth quarter of 2005.

Horizon(k)		1	2	3	4	5	6	7	8
<i>cay</i>	<i>cay</i>	<b>151.66</b>	<b>285.07</b>	<b>385.75</b>	<b>501.01</b>	<b>602.39</b>	<b>709.46</b>	<b>790.90</b>	<b>865.34</b>
		(48.11)	(82.72)	(121.4)	(153.2)	(172.3)	(104.4)	(204.7)	(210.6)
	$R^2$	5.00	8.50	9.87	12.77	14.93	17.70	19.37	21.43
<i>ch</i>	<i>ch</i>	<b>33.74</b>	<b>54.82</b>	<b>70.59</b>	<b>88.42</b>	<b>109.97</b>	<b>122.38</b>	<b>140.96</b>	<b>142.5</b>
		(9.49)	(17.89)	(21.65)	(26.68)	(19.56)	(21.07)	(38.24)	(37.53)
	$R^2$	6.30	7.45	8.40	10.19	12.67	13.48	16.20	14.91
<i>cay</i> and <i>ch</i>	<i>cay</i>	9.11	182.55	313.92	463.15	<b>493.07</b>	<b>708.69</b>	<b>700.65</b>	<b>980.62</b>
		(87.61)	(120.7)	(186.9)	(247.3)	(252.8)	(243.9)	(250.9)	(290.7)
	<i>ch</i>	<b>32.16</b>	23.11	16.19	24.52	24.7	0.17	20.29	-25.91
		(17.47)	(25.80)	(30.62)	(38.43)	(39.84)	(38.04)	(40.96)	(42.63)
	$R^2$	5.43	8.67	9.55	13.51	14.68	17.30	19.06	21.17

### Table-3 Predictability Using Residuals

This table reports estimates from OLS regression of market excess return on lagged *cay* and the residual from a regression of *ch* on *cay*. The market return here is the value weighted quarterly market returns from CRSP. *cay* is the ratio of consumption to wealth from Martin Lettau's website, *ch* is the ratio of nondurable consumption to housing for homeowners. Column 1-8 report the estimates for forecasting horizons ranging from 1 to 8 quarters. The Newey-West corrected standard errors appear in parentheses below the coefficient estimates. Panel A reports the estimates for market return, panel B reports the estimates for excess return. Significant estimates at 5% level are highlighted in bold face. Regressions use data from the fourth quarter of 1952 to the fourth quarter of 2005.

Panel A Predictability of Market Return

Horizon(k)		1	2	3	4	5	6	7	8
	<i>cay</i>	<b>148.95</b>	<b>285.89</b>	<b>388.69</b>	<b>503.28</b>	<b>600.54</b>	<b>704.77</b>	<b>779.65</b>	<b>854.01</b>
		(44.34)	(63.57)	(111.7)	(141.2)	(158.4)	(175.8)	(188.5)	(197.2)
<i>cay</i> and <i>resi</i>	<i>resi</i>	<b>39.67</b>	25.11	18.32	23.59	43.79	55.23	47.30	59.91
		(18.11)	(25.89)	(30.58)	(32.02)	(39.12)	(44.28)	(44.75)	(46.88)
	$R^2$	6.55	8.47	10.34	13.50	16.08	19.14	21.21	22.77

Panel B Predictability of Excess Return

Horizon(k)		1	2	3	4	5	6	7	8
	<i>cay</i>	<b>145.38</b>	<b>280.78</b>	<b>382.92</b>	<b>497.61</b>	<b>596.38</b>	<b>702.43</b>	<b>779.72</b>	<b>857.37</b>
		(45.63)	(86.20)	(119.9)	(151.7)	(170.7)	(187.9)	(200.6)	(113.8)
<i>cay</i> and <i>resi</i>	<i>resi</i>	<b>40.71</b>	27.25	21.58	26.53	46.31	57.47	75.34	61.24
		(18.14)	(29.03)	(30.15)	(31.01)	(37.79)	(42.61)	(43.26)	(45.55)
	$R^2$	6.27	7.94	9.64	12.59	15.09	18.05	20.10	21.73

**Table-4 Cross-Section Test Results**

This table reports the estimates of the slope coefficients for the cross-sectional regression of the form:  $r_i = r + \beta'\lambda$ , where *beta's* are obtained from time series regression of asset return on factors. The factors are value weighted market return ( $R_{vw}$  from CRSP), real estate returns (RRE from OFHEO), consumption growth rate (CG from NIPA tables), and Fama-French factors, SMB and HML from French's website. The *ch* factors are just the product of *ch* and the factors. The cross-sectional regression is ran for each period from 1952Q1-2005Q4, the standard errors are calculated by Fama-MacBeth method and corrected by Shanken's method. The *t*-statistics are reported below the coefficient estimates in parentheses. The reported  $R^2$ 's are time series average of  $R^2$ 's for each period. Significant estimates at the 5% level is highlighted in bold face.

	factors					ch factors			R <sup>2</sup>	
	<i>ch</i>	$R_{vw}$	RRE	CG	SMB	HML	$R_{vw}$	RRE		CG
1		0.83 (0.32)								0.02
2		-1.43 (-1.59)	<b>0.7</b> (2.64)							0.43
3				0.018 (1.86)						0.2
4		<b>4.17</b> (2.13)			<b>5.67</b> (2.97)	<b>2.25</b> (1.27)				0.68
5	-0.002 (-0.33)	<b>-1.79</b> (-2.04)					<b>2.87</b> (1.96)			0.56
6	0.23 (0.52)			<b>-0.013</b> (3.19)					0.01 (0.54)	0.34
7	0.005 (-1.52)	<b>3.82</b> (2.00)	-0.3 (-1.35)				<b>6.61</b> (2.06)	<b>-0.5</b> (-1.99)		0.65

Figure-1 Model Fitted Return *vs* Realized Return

